IDENTIFYING AND NURTURING MATHEMATICAL TALENT

Clarence F. Stephens

The problem of teaching mathematics effectively to undergraduate students is one which does not have easy solutions. In my opinion, the problem has no solution which is independent over time and place. This article describes some of my experiences in identifying and nurturing mathematical talent at different places and at different times. My hope is that a knowledge of these experiences may prove helpful to others who seek to recognize and to develop the mathematical talents of students of the kind which in the past has not been generally recognized and nurtured.

My more than 45 years of teaching experience has been about equally divided between historically Black colleges and historically white colleges. I began my teaching career in 1940 at Prairie View University in Prairie View, Texas. I spent 15 years -- beginning in 1947 -- as Chairman of the Mathematics Department at Morgan State University in Baltimore, Maryland. I joined the State University of New York in the Fall of 1962 at the College of Arts and Science at Geneseo. For the past 17 years, I have served as Chairman of the Mathematics Department of the College of Arts and Science at Potsdam.

Perhaps fewer that 10 Black scholars in the United States held the Ph.D. degree in mathematics when I began my teaching career in 1940. When I joined the State University of New York in 1962, its Colleges of Arts and Science were being established as liberal arts colleges from teachers colleges with the primary mission of the preparation of elementary teachers. At all four colleges where I have taught, the academic environment was not very favorable for identifying and nurturing mathematical talent when I joined the faculty. I believe that I am correct in writing that no graduate of any of the four colleges had earned a Ph.D. in mathematics by the time I joined the faculty, although one college had been established for over 100 years. Nevertheless, at each of these four colleges I taught a student, who entered the college as a freshman or was already enrolled at the college the year I joined the faculty, who later earned a doctorate. At three of these colleges the doctorate was a Ph.D. in mathematics, at Geneseo the doctorate was in mathematics education and at Morgan the doctorates were one in mathematics and one in mathematics education.

I concluded early in my teaching career that, regardless of time and place, I could teach mathematics effectively to most of my students only if I were successful in protecting and strengthening their self-esteem. A favorable environment for nurturing mathematical talent must be one that minimizes, both in the mathematics classroom and in the college environment, fear of mathematics and mathematics avoidance. Perhaps the only way such a favorable environment can be developed is by producing examples of many students who reach a high level of achievement in mathematics. As a general rule, these examples will be few at the beginning and these few successful examples will make it easier to produce more examples. The best way to start the production of these examples is from the group of students regularly admitted to the college.

Typically, colleges with few mathematics majors and no tradition for developing many students to a high level of achievement in mathematics assign their entering freshmen to remedial or elementary mathematics cour-

ses by placement examinations. During my first year as a college teacher I was assigned to teach a course in College Algebra. When I gave a diagnostic examination to the class, I learned that one student had mastered the material of the course as a high school student. I told the student that he had already earned a grade of A for the course and that I would give him special assignments which he could do independently. However, I requested that he attend class since I wanted to use him as a teaching assistant for the class although he was a freshman. I wanted to provide for my College Algebra class an example of a student who had mastered the course and who had graduated from the same high school as that from which many other students in the class had graduated. The student who was recognized as indicated later earned the Ph.D. degree in mathematics from the University of Michigan.

Some mathematics teachers believe that the best way to prepare students for successful study for advanced degrees in mathematics is to help them obtain a good background in undergraduate mathematics. Often these students study remedial or elementary courses as freshmen and most of their undergraduate study in mathematics is devoted to preparing them to take advanced undergraduate mathematics courses, although only a very few of these advanced courses are completed before these students graduate from college.

My experience is that the best way to prepare students for successful study for advanced degrees in mathematics is to teach them graduate courses as early as possible in their undergraduate studies. We were able to provide examples of Morgan State University students who could do well in first year graduate mathematics courses during their junior year in college. This first group of examples of Morgan's students participated in an Undergraduate Research Participation Program which I directed and which was supported by the National Science Foundation.

We selected 8 students for this Undergraduate Research Participation Project - 6 sophomores, one junior, and one senior. Four of the students were men (all sophomores) and 4 were women. Students who participated in this program studied graduate mathematics during the summer of 1962.

The three men in the summer project later earned a Ph.D. degree in mathematics and the other man a Ph.D. degree in the mathematical sciences; the four women earned Master's degrees (2 from The Johns Hopkins University, one from Ohio University, and one from the University of Wisconsin). In addition, one of the women earned a law degree. An examination of intelligence tests and high school records indicated that Morgan State University had been admitting for many years students with academic ability and preparation equivalent to that of this special group of 8 students.

Graduate study in mathematics was not offered at Prairie View University and Morgan State University during the time I was a member of the faculty of these two universities. However, we were able to teach graduate mathematics courses in a senior seminar course which was required for mathematics majors in our honors mathematics program. A Master's degree in secondary mathematics was being offered at the State University of New York at Potsdam when I joined the faculty. The mathematics requirement for this degree is not very strong.

At educational institutions where the highest degree offered is the Master's degree, undergraduates are not generally encouraged to select

graduate mathematics courses as soon as they are able to do so. This condition exists, particularly, at educational institutions with a long tradition as a teachers college. In order to identify and to provide a favorable academic environment which would encourage our most able mathematics majors to reach a high level of achievement in mathematics we established in 1970 a double degrees program. Our Double Degrees program in mathematics provides an opportunity for able students to work for the bachelor's and master's degrees at the same time. A student can complete this program in 4 years without attending summer school and will be awarded the bachelor of arts degree and the master of arts degree in mathematics upon graduation.

Students generally enter the Double Degrees program during the fall semester of their junior year. A very few enter the program the fall semester of their sophomore year. They begin the program by taking two graduate courses during the fall semester - Abstract Algebra based on Hungerford's "Algebra" or a similar textbook and Real Variables based on "Real Analysis" by Royden or a similar textbook. They register for the corresponding undergraduate courses in Modern Algebra and Advanced Calculus. We publish a pamphlet which explains in more detail our Double Degrees program.

Students who show good progress in the development of mathematical maturity are recommended by the mathematics faculty to elect our course in Theory of Sets. The purpose of this course is to develop to a high level the mathematical maturity of the students who elect this course as well as their ability to work independently. Students who achieve at a high level in this course are invited by the mathematics faculty to enroll in our Double Degrees program.

We learned that students who entered our Double Degrees program during the first few years after its establishment needed a great deal of encouragement and support from the mathematics faculty in order to minimize the fear of undergraduates who were taking graduate courses for the first time. In order to help in this matter I once used as a textbook in my Theory of Sets course the book "Topology" by Dugundji, using the content in the first two chapters of this book together with some supplementary materials on the uses of the Axiom of Choice and Zorn's lemma as the subject matter for the course. I made a note of the names of students who were successful in solving problems in the second chapter in the Topology book by Dugundji and later learned that a senior in this class was the first woman graduate of our college to earn a Ph.D. degree in pure mathematics.

Although the undergraduate students who were admitted to our Double Degrees program were better mathematics students than our regular graduate students, these undergraduate students did not recognize this fact. Then, a few years after the establishment of this program a young woman enrolled in my Theory of Sets course as a freshman and made the highest score on our Master's written comprehensive examination for the Master of Arts degree in Mathematics as a sophomore. We publicized her success. As a result we enroll each year one or two students who make a similar achievement as this young woman. Now our regular graduate students display some doubt about their ability to compete successfully in graduate mathematics courses with undergraduate students who are enrolled in our Double Degrees program, but regular graduate students are inspired and do reach a high level of achievement as a result of these successful examples.

As a general rule, my Theory of Sets course now contains a mix of students who are freshmen, sophomores, juniors, seniors, and first year graduate students. One year I decided to use as a textbook for this course the book "Introduction to Set Theory" by J. Donald Monk. When a senior in the class wanted to complain that the book was too difficult, he made a greater effort to understand the material in the book when he realized that the best student in the class who was achieving very highly in the course was a freshman. This senior later earned the Master of Arts degree in mathematics from Michigan State University. I am convinced that successful examples are needed in the kind of environment which will nurture mathematical talent.

Frequently, mathematics faculties compete eagerly for an opportunity to teach advanced mathematics courses at undergraduate colleges with few mathematics majors and where the instructional mission of the mathematics department is to teach preponderantly remedial or elementary service courses. If these advanced mathematics courses are not assigned on a basis of equality, faculty morale is depressed as a result of little opportunity for faculty development, which results in uninspired teaching and low achievement in mathematics by students who enroll in mathematics courses. Students are usually blamed for their low mathematics achievement. Such an environment is unfavorable for identifying and nurturing mathematical talent.

A policy in our department is that each member of our mathematics faculty will be given an opportunity to teach at least one advanced mathematics course each semester and is assigned most often the first choice of an advanced course. Each faculty member teaches across the mathematics curriculum. Often a teacher who is teaching one of our most advanced graduate mathematics courses is at the same time teaching our most elementary mathematics courses. The welfare of our students is the most important consideration in our department and students respond well to our mathematics program. The number of enrolled students at our college is about 4,000 and more than 500 of these students are mathematics majors. Our mathematics department ranks first relative to the number of majors and our computer science department ranks second. We have separate mathematics and computer science departments. About 200 mathematics majors were on the President's List for high academic achievement and more than 160 were members of the national honors society Pi Mu Epsilon in the 1984-85 academic year. During the decade 1973-1982 when the number of mathematics majors was decreasing in the United States our college graduated an average of 70 mathematics majors each year. The number of graduates of our college with a major in mathematics, during a decrease in our college enrollment, has increased as follows: 69 in 1982, 96 in 1983, 124 in 1984 and 184 in 1985, which is about 20% of the 1985 graduating class.

Some members of our mathematics faculty are members of the Association of Mathematics Teachers of New York State, which is a professional organization serving primarily high school teachers of mathematics. These faculty members give talks at meetings, serve on committees and direct summer workshops which are sponsored by the association. We publicize the successes of our majors and mathematics alumni through the publication of an annual newsletter. A copy of this newsletter is sent to every high school represented by a mathematics major in a given graduating class. Otherwise, members of our mathematics faculty make no special effort to recruit mathematics majors among the high schools or at our college.

As educators we believe often that we can predict the possible future achievement of students as a group based on their academic record as a major guide. We may be able to make a reasonable guess when we consider students as individuals when administrators, teachers and students become fixed in their way of doing things and level of expectation.

Some administrators and teachers continue to make maximum effort to create a favorable learning environment for their students, to believe in their students and care for them, and to inspire them through creative teaching and encouragement. Under these conditions we may not be very good at predicting the intellectual potential and future achievement of an individual student with only the past academic records as a guide.

Last academic year I selected the college transcripts of 10 students who graduated from our college with a major in mathematics over a 7 year period. I removed the names from these transcripts and other identification marks. Our college president, provost, dean and mathematics faculty were requested to identify the transcripts of the men and women in the group, to guess the high school averages of the students and predict the probable future academic achievement of the students based on their college academic records. They were not very successful in their guesses and predictions.

The lowest high school average of students in the group is 83 and the highest average is 98. The group contains 5 men and 5 women. The two students with high school averages of 83, 85 and the student with a high school average of 98 all earned 4.0 college cumulative averages (the highest possible) during their senior years. Also, the two students with high school averages of 83 and 85 have almost identical college cumulative averages as that of another student in the group of 10 with a high school average of 94. The range of the differences in these three college cumulative averages is 0.01 to 0.08.

The 4 students in this group of 10 with the lowest college cumulative averages, which includes the two students with the lowest high school averages of 83 and 85, have each earned a Ph.D. degree in the mathematical sciences and another student who ranks 4th from the top based on college cumulative averages has earned also a Ph.D. degree in mathematics. Thus, the total number of students in this group of 10 who have earned a Ph.D. is 5, three women and two men. Although the 3 students with the highest college cumulative averages have not earned a Ph.D. degree, they are following successful careers as industrial mathematicians and an engineer. Two of the 5 students who have earned a Ph.D. degree are industrial mathematicians and the other three who earned a Ph.D. degree are college and university professors. Finally, all respondents conjectured that each student in this group of 10 has the ability and college preparation to earn a Ph.D. degree.

We could form perhaps at least 20 mutually exclusive groups of 10 students who graduated from our college with a major in mathematics during the last 15 years whose undergraduate academic records would compare favorably with the undergraduate academic records of this special group under consideration. Many students have potential talents which they, most of their teachers and administrators do not recognize. If these students have an opportunity to study under favorable learning conditions and have the maximum support and encouragement of both teachers and administrators, many of these students will recognize their potential and reach a high level of

excellence in their academic work and personal lives.