

R. D. #1, Box 248
Chazy, NY 12921
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Dear Steve,

I hope that the new year find you to be healthy and that you had a pleasant Christmas with your family. I have wanted to write for a long time to let you know how I am developing.

When you sent me the letter about a year and a half ago in which you said that what you did was to help students learn to read mathematics with understanding, I was left with one huge question: How do you know when a student has read something with understanding? For the past year I have asked myself that question, and I have asked others. Until recently, nobody, including me, has given me an answer. But now I see that the answer is really very simple. You know that someone has read something with understanding if they can explain to you what they have read. The answer is so simple yet so hard to see.

I have been thinking about why it has been so hard for me to see. I have had 18 years of teaching (before 1987) in which my fundamental belief was that in order to get students to understand, you must make things simpler. During those 18 years I had to deal with students who, I honestly had to admit, did not understand what I was talking about. I was not effective in helping my students learn successfully, and I felt that I wasn't. So in an effort to become more successful, I tried to make things easier for them, and incidentally, expect less of them. So when I began to experiment with some of the teaching techniques that you folks at Potsdam used, it was a complete turnabout for me. I think it has taken me until now to finally shed the last of the dysfunctional idea that making things easier for students will lead to success as a teacher. Part of the problem is that making things harder for students just for the sake of making things harder doesn't work either.

The idea that students will learn to read best by explaining to others what they have read has always seemed to me to be the ultimate standard. After all, I have often said that you truly come to understand a subject when you to teach it. But whenever I said that, I always had the accompanying thought that I never completely understood what was in my graduate school courses and that by teaching the subject, I was able to fill out my understanding. In other words, in my own experience, complete understanding always came after I had finished a course, not while I was taking it. Thus I came to have the same expectations for my own students. A deep level of understanding thus was not something that I expected from my students, because I was seldom able to achieve it myself. (I am now looking at

this last sentence with complete astonishment. I have not realized until this moment how I have had two completely contradictory expectations for my students simultaneously.) Yet all my efforts as an educator were to try to engender a deep level of understanding in my students. This is amazing!

Anyway, I now realize that you understood long ago that students do not know how to read with understanding, and that one way to help them learn how to do so is by explaining proofs. I now realize that when I read a proof, I explain it to myself. I don't do the explaining on paper. I do it in my head. And until recently I have been unaware of the activity. No one ever taught me to do it or made me aware of it. I just learned that that is what I needed to do in order to be successful when I was in graduate school. You will recall that I attended your talk to the Seaway Section at Syracuse University a few years ago. When someone asked you at the end of your talk what you really did, you talked about the two-color proof. At the time I wondered about your answer. Now I know that you were really telling them the complete story, even though you did not make it sound that way..

This past spring I wrote to tell you what I was doing in the Sets, Functions and Relations course. At the beginning of the course I gave my students many examples of carefully worked out proofs, ones where the logic was very clear. At the end of the proof I would write what had been done in the proof and the technique that had been used. At the beginning, I would write out the entire proof and have them copy it. Later I began to ask them to supply reasons. I did a total of 19 of these "in class" proofs, one each class period for about half the semester. I talked about how we have to put definitions into iff form in order for them to be useful in proving theorems. And I made sure that they saw that when they didn't, they couldn't prove theorems. They would run into trouble sometimes when they didn't take the time to write their definitions carefully. I asked them to do four problem sets during the semester, and each problem set had at least one two-color proof on it. Typically, they would do the two-color proofs last because they found them to be the most difficult of all the problems and the ones that they liked to do the least. So I began to wonder if requiring the two-color proofs wasn't too high an expectation for them. At that point, I didn't yet have an answer to my question of how to tell when someone understands what they have read. Nevertheless, that semester gave me confidence that I really did understand for myself all the nuts and bolts that go into doing a proof.

The students liked the course because they felt that they understood everything that went on. One student wrote after the semester was over that I had put him through hell, but he thanked me for restoring his interest in mathematics. So the students liked what I did, even though a couple withdrew because

they saw that there would be too much work expected. One thing that disappointed me were some complaints that the final exam was too hard, even though I tried to make it very straightforward. And lately I have come to see that students from that course were unable to read with understanding proofs that their instructor gave them in Abstract Algebra this semester. Otherwise, it was an exhausting but satisfying experience. And it led me to try some new things.

Over the past few years I have taught a section of honors calculus. The Director of the Honor's Center wants honors courses to be done in seminar style, and of course I want to do calculus that way. Over these years, I have noticed that many students never really get proficient with algebra. They have had trouble doing algebra even into Calculus II. This has bothered me and I have wanted to do something about it. So I decided that last fall I would use "The Calculus with Analytic Geometry" by Louis Leithold. I knew that this book is very challenging for students, but I wanted to see if I could be successful in using it. I think that I tried to do too much, and didn't explain enough.

I didn't develop the sense of complete student understanding that I was able to create in Sets, Functions and Relations course last spring. So I am dissatisfied with what I did. I now believe that that level of understanding is very difficult to obtain in a course where coverage of material is very important. Anyway, even though I am trying to think of what to do next semester, there is one thing that I have learned from the course.

I have stretched my students. I have seen the strain on their faces. I am being very careful to make sure that students who do the minimal amount of work will pass the course with a C. And the other students who are working hard and contributing a lot will get an A. The thing I have learned is that I can have high expectations for my students, more than I ever have had before, and will have many of them respond well to the challenge. This has given me confidence to try something new next semester.

This coming spring I am scheduled to teach our course in Abstract Algebra. I believe that I will try to do it with a strong emphasis on having students do two-color proofs. I would like to have them to do two hand-in problems a week, one an original proof, and the other a two-color proof. Students have told me that they find doing them hard. But I now believe that if they can learn how to explain a proof to themselves, they will know all that they need to be successful with their future studies in mathematics. I will start slowly and will try to create the kind of complete understanding that my students talked about last spring. I will focus more on the long term goal of making sure my students know how to read mathematics and

less on the content of the course.

I had another experience last semester that has helped to shape my thinking. I taught a course entitled "Modern Mathematics for Elementary School Teachers I." I found a marvelous book that consists only of activities that involve hands-on experiences. For example, the activities involve working with small blocks and solving problems posed about them. Students must write extensions. These are variations of an activity that the student generates by asking the question "What would happen if I did" They must pose such a problem that they have thought up themselves and then solve it. The extension consists of writing up and explaining their solution. (There is that word "explain" again.) I have seen the benefits to students from doing this, and have become more convinced than ever that explaining what they do is an excellent activity for them.

I now believe that I understand how to tell when a student has read something with understanding, and why it is important for students to be able to do so. This new understanding has allowed me to use this idea as a standard to judge ideas for educational improvement. The question I ask myself is whether the idea will help students learn to read more effectively.

I will keep you informed of my progress and the kinds of results that I have with my students. I wish you a Happy New Year, and I hope that all is well with you. And I thank you very much for sending me such long letters. It has taken a lot of effort on your part, and even though I have not completely understood what you were talking about when I first got them, I now have a better understanding. Finally, I want to thank you for letting me send you my long letters. As I write this, I feel that you may be one of the few people alive today who could understand what I am talking about. So I appreciate being able to write to you. Thank you for all you have done for me.

With much gratitude,

Robert D. Hofer