

JULY 27, 1991

DEAR ROBERT,

BEST WISHES IN CARRYING OUT YOUR PLANS TO HELP YOUR STUDENTS UNDERSTAND WHAT A MATHEMATICAL PROOF IS AND HOW TO DEVELOP THEIR ABILITY TO GIVE CORRECT MATHEMATICAL PROOFS. I WILL BE INTERESTED IN THE RESULTS YOU OBTAIN AFTER SEVERAL YEARS OF EXPERIMENTING WITH YOUR PLANS.

EARLY IN MY TEACHING CAREER AND MANY YEARS BEFORE I JOINED THE MATHEMATICS FACULTY AT SUNY, I CONCLUDED THAT THE BEST WAY I COULD HELP STUDENTS TO WRITE CORRECT PROOFS WAS TO TEACH THEM TO READ MATHEMATICS TEXTBOOKS WITH UNDERSTANDING, PERHAPS ONE OF THE BEST THINGS A TEACHER CAN DO FOR STUDENTS. THIS TASK IS NOT EASY, BUT REQUIRES TIME, PATIENCE AND ENCOURAGEMENT OF STUDENTS. I OBSERVED THAT PROOFS ARE WRITTEN IN TEXTBOOKS AND TEACHERS EXPLAIN PROOFS IN THEIR LECTURES - HOWEVER, BOTH STUDENTS AND TEACHERS ARE OFTEN DISAPPOINTED WITH THE PROOFS GIVEN BY STUDENTS OF THEOREMS WHEN THEY HAVE NOT SEEN A PROOF OF THE THEOREMS BEFORE. I CONJECTURED THAT MOST STUDENTS ACCEPT PROOFS GIVEN IN TEXTBOOKS AND BY TEACHERS IN THEIR LECTURES ON AUTHORITY (THEY MEMORIZE THE PROOFS) AND NOT ON UNDERSTANDING. HENCE, THESE STUDENTS HAVE DIFFICULTY GIVING PROOFS ON THEIR OWN. ANY STUDENT WHO DEMONSTRATED THAT HE OR SHE COULD READ WITH UNDERSTANDING THE TEXTBOOK

FOR MY COURSE ALWAYS EARNED A GOOD GRADE IN MY COURSE, SINCE I KNEW THE STUDENT WOULD BE SUCCESSFUL IN FUTURE MATHEMATICS COURSES THE STUDENT STUDIED, REGARDLESS OF HOW GOOD OR BAD HIS OR HER FUTURE TEACHERS WOULD BE.

I HAD SOME INTERESTING EXPERIENCES ABOUT THE TASK OF READING A MATHEMATICS TEXTBOOK WITH SOME OF MY COLLEAGUES AT TWO DIFFERENT COLLEGES WHERE I TAUGHT BEFORE I JOINED THE MATHEMATICS FACULTY AT SUNY POTSDAM. THERE WAS A TOTAL OF FOUR PERSONS INVOLVED ON DIFFERENT OCCASIONS. EACH OF THESE COLLEAGUES HAD EARNED A MASTER'S DEGREE IN MATHEMATICS AT UNIVERSITIES WITH STRONG RESEARCH MATHEMATICS DEPARTMENTS, AFTER TWO YEARS OF GRADUATE STUDIES. EACH WAS CONSIDERING PLANS TO CONTINUE HIS OR HER STUDIES FOR THE Ph.D. DEGREE IN MATHEMATICS. I WANTED TO READ THE FOLLOWING BOOKS:

- (1) GENERAL TOPOLOGY BY JOHN L. KELLEY
D. VAN NOSTRAND AND COMPANY, INC., 1955
- (2) LECTURES IN ABSTRACT ALGEBRA BY
NATHAN JACOBSON, VOLUME 1
D. VAN NOSTRAND AND COMPANY, INC. 1951
- (3) FUNDAMENTAL CONCEPTS OF ALGEBRA
BY CLAUDE CHEVALLEY, ACADEMIC PRESS, INC. 1956
- (4) TOPOLOGICAL VECTOR SPACES BY JOHN HORVÁTH
LECTURE NOTES, NUMBER 2, 1963
DEPARTMENT OF MATHEMATICS, UNIVERSITY
OF MARYLAND

ON EACH OCCASION I AGREED TO LEAD THE DISCUSSION AT THE BEGINNING AND THAT WE WOULD ALTERNATE AS LEADERS. I SOON LEARNED THAT MY COLLEAGUES COULD NOT LEAD THE DISCUSSION SINCE THEY COULD NOT READ THE TEXTBOOK, ALTHOUGH THEY HAD COMPLETED GRADUATE COURSES IN ALGEBRA AND TOPOLOGY. HENCE, I LED THE DISCUSSIONS AND WE COMPLETED ABOUT HALF OF EACH BOOK. EACH OF THESE COLLEAGUES TOLD ME AFTER OUR STUDY THAT THIS WAS THE FIRST TIME THEY HAD LEARNED HOW TO READ A MATHEMATICS TEXTBOOK. EACH RETURNED TO GRADUATE SCHOOL, EARNED A'S IN HIS OR HER GRADUATE COURSES, WAS AWARDED THE Ph.D. IN MATHEMATICS, ALTHOUGH ALMOST ALL OF THEIR GRADES WERE B'S WHEN THEY STUDIED FOR THE MASTER'S DEGREE.

WE TAUGHT DR. CATEFORIS, NOW CHAIR OF SUNY AT POTSDAM MATHEMATICS DEPARTMENT, HOW TO READ A MATHEMATICS TEXTBOOK WITH UNDERSTANDING AND THE MEANING OF A MATHEMATICS PROOF, WHEN HE WAS AN UNDERGRADUATE STUDENT AT MORGAN STATE COLLEGE. HENCE, HE WAS SUCCESSFUL IN HIS STUDIES FOR THE Ph.D. IN MATHEMATICS AT THE UNIVERSITY OF WISCONSIN, AT MADISON. FOR EXAMPLE, HE WAS A MEMBER OF MY MODERN ALGEBRA CLASS WHEN I WAS TEACHING AT MORGAN STATE COLLEGE. THE TEXTBOOK FOR THE COURSE WAS LECTURES IN ABSTRACT ALGEBRA BY NATHAN JACOBSON, VOLUME I, D. VAN NOSTRAND AND COMPANY, INC., 1951, ALTHOUGH NO

DEPARTMENT OFFERED A GRADUATE PROGRAM DURING THE TIME I SERVED ON THE MATHEMATICS FACULTY AT MORGAN STATE COLLEGE (NOW MORGAN STATE UNIVERSITY). DR. CATEFORIS DEMONSTRATED IN MY COURSE THAT HE HAD ACHIEVED AT A HIGH LEVEL THE GOALS OF MY COURSE.

ALSO, DR. CATEFORIS WAS ONE OF MY ASSISTANTS WHEN I DIRECTED AN UNDERGRADUATE SCIENCE EDUCATION PROGRAM, SPONSORED BY THE NATIONAL SCIENCE FOUNDATION, MORGAN STATE COLLEGE, BALTIMORE, MARYLAND, SUMMER 1962. EIGHT STUDENTS WERE ^{IN} THE PROGRAM, 4 MEN AND 4 WOMEN. SIX OF THE STUDENTS HAD COMPLETED THEIR SOPHOMORE YEAR AT MORGAN STATE COLLEGE DURING THE SPRING OF 1962. ALL OF THE MEN WERE SOPHOMORES DURING THE ACADEMIC YEAR 1961-62 AND TWO OF THE WOMEN WERE SENIORS, ONE OF THEM BEING A GRADUATING SENIOR. SOPHOMORES IN THE SUMMER PROJECT WERE EACH GIVEN ASSIGNMENTS TO READ INDEPENDENTLY ONE OF THE FOLLOWING BOOKS:

- (1) REAL ANALYSIS BY EDWARD JAMES MCSHANE AND TRUMAN BOTTS, D. VAN NOSTRAND INC., 1959, CHAPTERS I, II, III, PAGES 1-97.
- (2) COMPLEX ANALYSIS BY LARS U. AHLFORS, MCGRAW-HILL BOOK CO., INC. 1953, CHAPTERS I AND II, PAGES 1-81.

- (3) MATHEMATICAL ANALYSIS
BY TOM APOSTOL, ADDISON-WESLEY PUBLISHING
CO., 1957, CHAPTERS 1-6, PAGES 1-126.
- (4) ANALYTIC FUNCTIONS BY ELNAR HILLE
- (5) AN INTRODUCTION TO ALGEBRAIC TOPOLOGY
BY ANDREW H. WALLACE, PAGES 1-89,
PERGAMON PRESS, 1957
- (6) THEORY OF FUNCTIONS OF A REAL VARIABLE
BY EDWIN HEWITT, CHAPTERS I AND II,
PAGES 1-98.

ALL OF THE WOMEN IN THE SUMMER PROGRAM LATER EARNED MASTER'S DEGREES AND EACH OF THE MEN EARNED THE PH.D. DEGREE, 3 IN MATHEMATICS AND THE OTHER IN THE MATHEMATICAL SCIENCES, WITH EMPHASIS ON COMPUTER SCIENCE. DR. SCOTT WILLIAMS, PROFESSOR OF MATHEMATICS AT SUNY BUFFALO, WAS A STUDENT IN THE INDICATED SUMMER PROGRAM, AND HE IS ONE OF THE CO-AUTHORS OF AN INTERVIEW WITH CLARENCE STEPHENS, WME TRENDS, VOLUME 2, NUMBER 1, MARCH 1990.

WHEN I JOINED THE SUNY AT POTSDAM MATHEMATICS FACULTY IN THE FALL OF 1969, PERHAPS NO GRADUATE OF SUNY AT POTSDAM HAD EARNED THE PH.D. IN MATHEMATICS, ALTHOUGH THE COLLEGE WAS FOUNDED IN 1816. TO THE BEST OF MY KNOWLEDGE, DR. ARNOLD M. DUNN IS THE ONLY GRADUATE OF SUNY POTSDAM BEFORE 1970 WHO LATER EARNED THE PH.D. IN MATHEMATICS. HE WAS A TENURED MEMBER OF THE SUNY AT POTSDAM MATHEMATICS DEPARTMENT WHEN I JOINED THE FACULTY IN 1969 AND HE EARNED THE PH.D. DEGREE SEVERAL YEARS AFTER 1969 AT CLARKSON COLLEGE OF TECHNOLOGY IN POTSDAM, N.Y. IF ANY GRADUATE OF SUNY AT POTSDAM BEFORE 1970 EARNED THE PH.D. IN MATHEMATICS, OTHER THAN DR. DUNN, THIS KNOWLEDGE SEEMS TO BE UNKNOWN TO THE FACULTY AND ADMINISTRATION OF THE COLLEGE. HENCE, THE ACHIEVEMENT OF SUCH A POSSIBLE GRADUATE COULD NOT BE USED AS A ROLE MODEL TO HELP INSPIRE A HIGH LEVEL OF ACHIEVEMENT IN MATHEMATICS AT SUNY POTSDAM.

SUNY AT POTSDAM WAS OFFERING THE MASTER OF SCIENCE IN SECONDARY MATHEMATICS WHEN I JOINED THE FACULTY IN 1969. THIS DEGREE REQUIRED ONLY 9 SEMESTER HOURS CREDITS IN MATHEMATICS AND THE COURSES IN MATHEMATICS WERE AT THE LEVEL OF UPPER LEVEL UNDERGRADUATE MATHEMATICS COURSES. WE DID OBTAIN APPROVAL OF A MASTER OF ARTS DEGREE IN MATHEMATICS DURING 1969-70, MY FIRST YEAR ON SUNY AT POTSDAM FACULTY. HOWEVER, WE DID NOT HAVE FUNDS FOR GRADUATE ASSISTANTSHIPS IN ORDER TO ATTRACT ABLE GRADUATE STUDENTS

TO ENROLL IN THE MASTER OF ARTS DEGREE PROGRAM AND UNDERGRADUATES WERE REQUIRED TO BE WITHIN 9 SEMESTER HOURS OF COMPLETING THEIR BACHELOR'S DEGREE BEFORE THEY COULD ENROLL IN GRADUATE COURSES IN ANY SUBJECT AT SUNY POTSDAM.

THUS, BY THE ACADEMIC YEAR 1969-70 THERE HAD NOT BEEN ESTABLISHED AT SUNY POTSDAM A TRADITION FOR SOPHOMORES AND JUNIORS STUDYING GRADUATE MATHEMATICS COURSES. IN FACT, SOPHOMORES AND JUNIORS WERE NOT PERMITTED TO ENROLL IN GRADUATE COURSES. WHEN I JOINED THE FACULTY AT SUNY POTSDAM, I FOUND 6 TENURED MEMBERS OF THE MATHEMATICS FACULTY AND ONLY 2 OF THEM HAD BEEN AWARDED DOCTORAL DEGREES, ONE IN MATHEMATICS EDUCATION AND ONE IN PHYSICS. THE 2 TENURED MEMBERS WITH DOCTORAL DEGREES HAD COMPLETED THEIR UNDERGRADUATE STUDIES IN ENGINEERING PROGRAMS. THREE OF THE FOUR TENURED MEMBERS WERE AWARDED DOCTORAL DEGREES AFTER 1970 AND THE OTHER COMPLETED ALL REQUIREMENTS FOR THE Ph.D. IN MATHEMATICS, EXCEPT THE DISSERTATION. WHEN I VISITED WITH HIS FORMER MAJOR ADVISER, HE STATED THAT HE GAVE THIS TENURED FACULTY MEMBER A PROBLEM WHICH WAS TOO DIFFICULT AND THE PROBLEM HAD NOT BEEN SOLVED WHEN HE DISCUSSED THIS MATTER WITH ME LAST YEAR. ONE MEMBER OF THE MATHEMATICS FACULTY WITH A Ph.D. IN MATHEMATICS

JOINED THE FACULTY ONE YEAR BEFORE I DID AND TWO MEMBERS WITH A Ph.D. IN MATHEMATICS JOINED THE FACULTY THE SAME YEAR I DID. HENCE, LESS THAN HALF (6 MEMBERS) OF THE 13 MEMBERS MATHEMATICS FACULTY HELD THE DOCTORAL DEGREE DURING THE ACADEMIC YEAR 1969-70. WHEN I PROPOSED OUR BA/MA OR MS ^{FACULTY} PROGRAMS SOME MEMBERS OF THE MATHEMATICS FACULTY HAD GREAT DOUBTS ABOUT THE ABILITY OF ANY OF OUR STUDENTS BEING SUCCESSFUL IN SUCH A DEMANDING PROGRAM. SINCE I WAS SERVING AS CHAIR OF OUR MATHEMATICS DEPARTMENT, I WAS ABLE TO PERSUADE A MAJORITY OF THE MEMBERS OF MATHEMATICS ^{FACULTY} TO APPROVE OUR BA/MA OR MS PROGRAMS.

ONCE THE MA AND MS MATHEMATICS PROGRAMS HAD BEEN APPROVED, THE ESTABLISHMENT OF THE BA/MA OR MS PROGRAMS REQUIRED NO NEW COURSES OR ADDITIONS TO OUR FACULTY. ALL THAT WAS NEEDED WAS A RULE WHICH WOULD PERMIT STUDENTS BELOW THE SENIOR LEVEL TO ENROLL IN GRADUATE COURSES. SEE THE ENCLOSED PAMPHLET

FOUR-YEAR BACHELOR'S - MASTER'S DEGREES PROGRAM IN MATHEMATICS

NEVERTHELESS, I HAD CONSIDERABLE DIFFICULTY OBTAINING THE APPROVAL OF THE COLLEGE FACULTY FOR OUR BA/MA OR MS PROGRAMS. MANY MEMBERS OF THE COLLEGE FACULTY BELIEVED THAT WE DID NOT ENROLL ANY STUDENTS WHO COULD BE SUCCESSFUL IN SUCH A DEMANDING PROGRAM. IN ORDER TO HELP PERSUADE THE COLLEGE FACULTY TO APPROVE OUR BA/MA OR MS PROGRAMS, I PREPARED THE PAMPHLET

UNDERGRADUATES IN GRADUATE MATHEMATICS COURSES?

A COPY OF WHICH IS ENCLOSED. THE MATHEMATICS DEPARTMENT WAS THE ONLY DEPARTMENT AT THE COLLEGE FOR A PERIOD OF MORE THAN 10 YEARS THAT OFFERED THE BA/MA DEGREE PROGRAM. I BELIEVE THAT THIS IS ONE OF THE REASONS WE ATTRACTED SOME VERY ABLE STUDENTS TO SUNY POTSDAM AND MANY OF THEM MAJORED IN MATHEMATICS; WE HAD A VERY DEMANDING PROGRAM (BA/MA) IN WHICH STUDENTS SUCCEEDED.

MY EXPERIENCES AT MORGAN STATE COLLEGE GAVE ME CONFIDENCE THAT WE REGULARLY ADMITTED STUDENTS WHO COULD BE SUCCESSFUL IN OUR BA/MA OR MS PROGRAMS. WHEN I MADE THE PROPOSAL FOR OUR BA/MA PROGRAM, I KNEW THAT I HAD DEVELOPED 3 YOUNG WOMEN IN MY CALCULUS III CLASS DURING 1962-70 WHO COULD SUCCEED IN THIS PROGRAM. THE TEXTBOOK FOR MY CALCULUS III COURSE WAS

CALCULUS OF VECTOR FUNCTIONS

BY
RICHARD E. WILLIAMSON, RICHARD H. CROWELL
AND HALE F. TROTTER, SECOND EDITION,
PRENTICE-HALL, INC. 1962, 1968.

THESE 3 STUDENTS DEMONSTRATED THAT THEY COULD READ SECTIONS OF THIS BOOK INDEPENDENTLY AND THEY WERE THE FIRST STUDENTS WHO COMPLETED OUR BA/MA OR MS PROGRAMS IN 1972.

YOU MAY BE INTERESTED IN THE FACT THAT I WAS NOT SUCCESSFUL IN PREPARING A FOURTH WOMAN IN MY CALCULUS III CLASS FOR OUR BA/MA PROGRAM. AT THE BEGINNING OF MY COURSE WHEN I GAVE SOME EXPLANATION OF CONCEPTS AND STUDENTS GAVE PRESENTATIONS IN CLASS, SHE COMPETED VERY WELL WITH THE OTHER THREE STUDENTS. THEN I GAVE EACH OF THE FOUR STUDENTS INDIVIDUAL SECTIONS OF THE TEXTBOOK TO READ WHICH THE CLASS HAD NOT REACHED. I HELD CONFERENCES WITH EACH STUDENT TO DETERMINE THE PROGRESS THE STUDENT WAS MAKING IN CARRYING OUT THE ASSIGNMENT. AT THE TIME OF THE FIRST CONFERENCE WITH EACH STUDENT, THREE OF THE STUDENTS WERE MAKING GOOD PROGRESS IN COMPLETING THEIR INDIVIDUAL ASSIGNMENTS, BUT THIS FOURTH STUDENT HAD MADE NO PROGRESS. HER EXCUSE WAS THAT SHE COULD NOT STUDY THIS ASSIGNMENT SINCE SHE HAD UNUSUALLY HEAVY WORKLOADS IN HER OTHER COURSES.

WHEN I HEARD HER EXCUSE, I CONCLUDED THAT PERHAPS SHE WAS AN OVERACHIEVER IN THE FIRST PART OF MY

COURSE AND I HAD GIVEN HER A TOO DIFFICULT ASSIGNMENT. I DECIDED TO EXAMINE HER HIGH SCHOOL ACHIEVEMENT AND SAT SCORES AND COMPARE THESE MEASURES OF ACADEMIC PROMISE WITH THE OTHER THREE STUDENTS' MEASURES OF ACADEMIC PROMISE. USUALLY I DO NOT SEEK THESE MEASURES OF ACADEMIC PROMISE OF MY STUDENTS, SINCE I BELIEVE THAT ALL OF MY STUDENTS CAN REACH A HIGH LEVEL OF ACHIEVEMENT. TO MY SURPRISE, I LEARNED THAT THIS FOURTH STUDENT HAD MORE ACADEMIC PROMISE THAN THE OTHER THREE STUDENTS ACCORDING TO THE ABOVE INDICATED MEASURES. IN FACT SHE WAS ONE OF THE MOST PROMISING STUDENTS AT THE COLLEGE. I WAS COMPLETELY PUZZLED. AT OUR SECOND CONFERENCE THIS FOURTH STUDENT ENLIGHTENED ME. SHE EXPLAINED THAT SHE HAS A PHOTOGRAPHIC MEMORY. HER HIGH SCHOOL AND ALMOST ALL OF HER COLLEGE TEACHERS LECTURED AND EXPLAINED CONCEPTS. HENCE, SHE NEVER HAD TO STUDY IN HIGH SCHOOL OR IN COLLEGE IN ORDER TO PASS HER COURSES. SHE TOLD ME THAT IN ORDER TO COMPLETE THE INDIVIDUAL ASSIGNMENT I GAVE HER SHE WOULD BE REQUIRED TO STUDY AND TO THINK, TWO ACTIVITIES SHE HAD NOT PRACTICED.

THEN SHE SAID THAT SHE ENJOYED GOING TO SCHOOL WITHOUT STUDYING, AND SINCE MOST OF HER TEACHERS WOULD LECTURE, SHE WAS CONFIDENT THAT SHE COULD GRADUATE FROM COLLEGE WITHOUT STUDYING. I RELIEVED HER FROM THE INDIVIDUAL ASSIGNMENT I HAD GIVEN HER. SHE WAS AN INTELLIGENT STUDENT, BUT GRADUATED FROM COLLEGE WITH A LITTLE LESS THAN 3.0 COLLEGE CUMULATIVE AVERAGE.

PERHAPS YOU MAY BE INTERESTED IN ONE OF MY STUDENTS WHO IS ALMOST THE OPPOSITE IN LEARNING HABITS FROM THE FOURTH STUDENT DESCRIBED ABOVE. I TAUGHT THIS STUDENT THE LAST TWO YEARS BEFORE MY RETIREMENT. HE WAS A TRANSFER STUDENT FROM A COMMUNITY COLLEGE. WHEN HE CAME TO POTSDAM, HE ELECTED PHYSICS AS HIS MAJOR. HIS GRADES FROM THE COMMUNITY COLLEGE WERE ABOVE AVERAGE, BUT NOT OUTSTANDING. HE DID NOT ACHIEVE WELL IN HIS PHYSICS COURSES AT SUNY POTSDAM, BUT HIS ACHIEVEMENT IN HIS MATHEMATICS COURSES WAS BETTER THAN HIS ACHIEVEMENT IN PHYSICS COURSES, BUT NOT OUTSTANDING.

I FIRST TAUGHT HIM IN MY LINEAR ALGEBRA CLASS. I BEGAN TO TEACH STUDENTS IN THIS CLASS HOW TO READ THE TEXTBOOK BY SELECTING A STATEMENT IN THE PROOF OF A THEOREM WHICH IS NOT CLEAR. THE HOME WORK ASSIGNMENT WAS TO WRITE THE GIVEN STATEMENT IN ONE COLOR PENCIL AND THE STUDENT'S EXPLANATION OF HOW THAT PARTICULAR STATEMENT FOLLOWED FROM PREVIOUS STATEMENTS IN THE PROOF OF THE THEOREM IN ANOTHER COLOR PENCIL. I COLLECTED THE HOMEWORK ASSIGNMENTS AND READ THE PAPERS, MADE A NOTE OF THE PROGRESS OF STUDENTS IN MY RECORD BOOK, AND RETURNED THE PAPERS WITHOUT GRADING THEM. I HAD EXPECTED ABOUT 25 STUDENTS FOR THE CLASS, BUT 59 STUDENTS ENROLLED IN THE COURSE. HENCE, I HAD THEM WORKING IN GROUPS OF 7 OR 8. WHEN I RETURNED THE FIRST HOME WORK ASSIGNMENT, I TOLD THE MEMBERS OF THE CLASS THAT THEY DID NOT AGREE ON THE EXPLANATION FOR THE STATEMENT. THEY WERE TO WORK IN THEIR GROUPS IN CLASS AND CONVINCING THE MEMBERS OF THEIR GROUP THAT THEY HAD A GOOD EXPLANATION HOW

THE GIVEN STATEMENT FOLLOWED FROM THE PREVIOUS STATEMENTS IN THE PROOF OF THE THEOREM. WHEN STUDENTS OBSERVED THAT THEY DID NOT AGREE WITH EACH OTHER AND HAD DIFFICULTY REACHING A COMMON AGREEMENT, THEY REALIZED THAT THEY DID NOT KNOW HOW TO READ A MATHEMATICS TEXTBOOK. I CONTINUED THIS PROCESS GRADUALLY THROUGHOUT THE COURSE.

THIS TRANSFER STUDENT HAD NO IDEA OF HOW TO READ HIS TEXTBOOK AT THE BEGINNING OF THE COURSE, BUT HE BECAME VERY GOOD AT READING THE TEXTBOOK BEFORE THE COURSE WAS OVER. HENCE, I INVITED HIM TO ENROLL IN MY THEORY OF SETS COURSE. THE TEXTBOOK FOR THE COURSE WAS SET THEORY BY CHARLES C. PINTER, ADDISON-WESLEY PUBLISHING CO. (I USED MANY DIFFERENT TEXTBOOKS FOR THIS COURSE, BUT I WAS NOT SATISFIED WITH ANY OF THEM FOR MY CLASS. I SHOULD HAVE WRITTEN A TEXTBOOK FOR MY COURSE.) I GAVE THE CLASS THE HOMEWORK ASSIGNMENT TO EXPLAIN IN DETAIL THE THEOREM

8.11 THEOREM (PAGE 159) IF B IS AN INFINITE CARDINAL NUMBER, THEN

$$B^B = B$$

(WE MAY NOTE THAT THIS CLASS ENROLLED FRESHMEN, SOPHOMORES, JUNIORS, SENIORS AND GRADUATE STUDENTS.)

IN PARTICULAR, THEY WERE TO PAY SPECIAL ATTENTION TO THE STATEMENT;

NOW IT IS EASY TO VERIFY THAT A SATISFIES THE HYPOTHESES OF ZORN'S LEMMA (THE DETAILS ARE LEFT AS AN EXERCISE FOR THE READER).

THIS PARTICULAR TRANSFER STUDENT DID AN EXCELLENT JOB OF WRITING THE DETAILS OF THIS PROOF AND I KNEW THAT HE UNDERSTOOD THIS PROOF. ALSO, I KNEW THAT HE HAD REACHED THAT LEVEL OF MATHEMATICAL MATURITY SO THAT HE WOULD BE SUCCESSFUL IN ANY FUTURE MATHEMATICS COURSE HE STUDIED REGARDLESS OF HOW GOOD OR BAD HIS FUTURE TEACHERS WOULD BE. ALTHOUGH HE DID ^{NOT} HAVE A COLLEGE CUMULATIVE AVERAGE OF AT LEAST 3.0, I INVITED HIM TO ENROLL IN OUR BA/MA PROGRAM.

IN THE FALL OF THE YEAR WHEN HE COMPLETED MY THEORY OF SETS COURSE IN THE SPRING OF THAT YEAR, HE WAS ENROLLED IN OUR COURSES:

(1) ABSTRACT ALGEBRA, TEXTBOOK: ALGEBRA BY THOMAS W. HUNGERFORD, HOLT, RINEHART AND WINSTON, INC. 1974

(2) REAL VARIABLES, TEXTBOOK: REAL ANALYSIS BY H. L. ROYDEN, THE MACMILLAN COMPANY, 1968. THESE CLASSES MET TWICE A WEEK AND HE WOULD ATTEND CLASS ABOUT ONCE EVERY TWO WEEKS. MY COLLEAGUE WHO WAS TEACHING THE REAL VARIABLES COURSE INFORMED ME OF THIS SITUATION AND SAID THAT WE HAD MADE A MISTAKE BY PERMITTING HIM TO ENROLL IN HIS CLASS SINCE HE BELIEVED THE COURSE WAS TOO DIFFICULT FOR THIS TRANSFER STUDENT. I TOLD MY COLLEAGUE THAT I BELIEVED THAT THIS TRANSFER STUDENT COULD READ REAL ANALYSIS BY ROYDEN ON HIS OWN. MY COLLEAGUE WAS OF THE OPINION THAT I WAS JOKING. ABOUT TWO MONTHS LATER, HE TOLD ME THAT THIS STUDENT HAD SOLVED SO MANY EXERCISES OUT OF ROYDEN'S BOOK THAT HE DID NOT HAVE TIME TO READ ALL OF THEM. I TOLD HIM THAT IF HE READ ONE OF HIS PROOFS WHICH HE BELIEVED TO BE INCORRECT, HE SHOULD LOOK AT THE PROOF CAREFULLY, SINCE THIS TRANSFER STUDENT ALMOST NEVER WROTE AN INCORRECT PROOF.

MY COLLEAGUE WHO WAS TEACHING OUR ABSTRACT ALGEBRA COURSE WAS A NEW TEACHER AND HE LECTURED IN HIS CLASS. ON ONE OF THE OCCASIONS WHEN THIS STUDENT ATTENDED HIS CLASS, HE INQUIRED OF THIS STUDENT AFTER CLASS IF HE UNDERSTOOD HIS LECTURE. THE STUDENT RESPONDED THAT WHEN A TEACHER LECTURES ALL THE TIME, HE TRIES NOT TO HEAR WHAT THE TEACHER IS SAYING SINCE THE TEACHER INTERFERES WITH HIS THINKING. HE RECEIVED GRADES OF A^(4.0) IN THESE TWO COURSES.

AFTER HIS SUCCESS IN OUR BA/MA PROGRAM, WHICH HE COMPLETED IN THE FALL SEMESTER, HE EXPRESSED A DESIRE TO STUDY FOR THE PH.D. IN MATHEMATICS. HIS COLLEGE CUMULATIVE AVERAGE WAS NOW A LITTLE ABOVE 3.0, BUT WE WERE ABLE TO PERSUADE THE CHAIR OF THE MATHEMATICS DEPARTMENT AT SUNY BINGHAMTON TO AWARD HIM AN ASSISTANTSHIP, WHEN THE CHAIR CAME TO SUNY POTSDAM TO LECTURE AND RECRUIT GRADUATE STUDENTS. WHEN THE CHAIR VISITED OUR COLLEGE THE FOLLOWING YEAR, HE TOLD US THAT THIS TRANSFER STUDENT WAS THE BEST STUDENT IN HIS CLASS. THE LAST INFORMATION I HAVE ON THIS STUDENT IS THAT HE IS DOING RESEARCH FOR HIS PH.D. IN MATHEMATICS.

STUDENTS RESPOND TO TEACHERS IN DIFFERENT WAYS. ONE OF THE STUDENTS DISCUSSED ABOVE WANTED HER TEACHERS TO LECTURE AND THE OTHER DID NOT.

IN ONE OF MY THEORY OF SETS CLASSES, I USED SET THEORY BY PINTER AS A TEXTBOOK. I TAUGHT MY STUDENTS HOW TO READ CHAPTER I OF THE TEXTBOOK. MY STUDENTS READ CHAPTER II WITHOUT MY HELP AND SOLVED ALMOST ALL THE EXERCISES IN CHAPTER II. MY JOB WAS TO POINT OUT SOME CORRECTIONS IN THE TEXTBOOK CHAPTER AND TO ACT AS MODERATOR WHEN STUDENTS PUT PROOFS OF EXERCISES ON THE BLACKBOARD.

FINALLY, YOU MAY BE INTERESTED IN THE OPINIONS OF TWO SUNY POTSDAM MATHEMATICS ALUMNI AS RECORDED IN THE FOLLOWING TWO ENCLOSED PAMPHLETS:

1. THOUGHTS ON THE REAL VALUE OF A MATHEMATICS EDUCATION BY SUE A. BAUER
 - (a) VALUE OF READING A MATHEMATICS TEXTBOOK, PAGE 4
 - (b) EXPRESS ONESELF VERBALLY AND IN WRITING, PAGES 5
 - (c) RIGOROUS MATHEMATICAL BACKGROUND, PAGE 3
2. WHY DO MATHEMATICS? BY DR. KATHRYN WELD
 - (a) REASON FOR CHANGE OF MAJOR, PAGE 1.
 - (b) ENROLLED IN A CALCULUS CLASS, PAGE 1.
 - (c) PREPARATION FOR PH.D. STUDIES, PAGE 4.

GOOD LUCK,

SINCERELY,
Steve

CLARENCE F. STEPHENS

P.S. SEE PAGE 22 OF THE 1985 NEWSLETTER WHICH SHOWS THAT 127 MATHEMATICS MAJORS WERE NAMED TO THE PRESIDENT'S LIST. OUR STUDENTS HELPED US TO TEACH OUR STUDENTS.