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Dear Steve,

I have been wanting to write you a letter for some time, and I have known that I would, but I find it hard to write for some reason. During the past semester I have had a crystallizing experience, an aha! experience. I think that I clearly understand why students are having trouble with proofs, and I think that I know what I can say to them to help them. In fact, the feeling I have about this is the kind of "perfect certainty" that Henri Poincare described when one has solved a problem that one has been working on for a long time.

I have been teaching our beginning proofs course (the analogue of Potsdam's Set Theory and Logic) for some time now, and have watched students struggle with the material, and then go on and have real trouble with Abstract Algebra. Some who have done well with the calculus courses have failed Abstract Algebra after having done the beginning proofs course with me. Needless to say, this has been a real frustration for me. Of course, a few also have done very well. But the numbers of successful students have not been nearly high enough for me. So I have felt very dissatisfied.

Anyway, I finally have realized a whole bunch of things. First of all, there are a number of things that most mathematics instructors expect their students to pick up by themselves simply because when they were students themselves, that is what they had to do. For example, somewhere along the line we all learned that maybe 90% writing a proof is basically a game of using replacement rules, except that we replace one statement with another, whereas in algebra we use expressions. We also learned somehow that definitions were the most important replacement rules, and we learned how to generate replacement rules from definitions of sets. My problem with teaching has been that I have been asking students to play a certain game without their knowing how to get the pieces they need to play with. So I have decided that this fall, when I teach our proofs course again, I will be talking a lot about this kind of thing at the beginning. I will be telling them how the game is played. But once I have described the game to them, I will expect them for the most part to be able to play it by themselves.

I expect that I may have a large number of successful students. And I also expect that they will be well prepared to deal with Abstract Algebra. I hope that they will be able to impress their later instructors so that the credibility of the method of teaching that I am using is greatly enhanced. I hope that the rest of the department will like my students.

When I first came to visit you at Potsdam in 1982, I sensed that your department was doing something that somehow made things easier for your students. I now know that what you did was to tell them how the game is played, what it is played with, and how to get the pieces you need to play it. But you also insisted that as much as possible, they play the game by themselves. So you did make things easier in a certain way, but not in a way that most people would be able to understand. And I suspect that your understanding of this came from watching your students struggle with proofs, and caring enough to be pained when they were not successful.

I also understand that all of this is very subtle. I had a chance to talk with Hector Foisy last fall at a Seaway Section meeting, and he told me that when you came to Potsdam, you really did nothing. He claims that the department there would have developed the way it did anyway. He feels that all it needed was some stable leadership, and he thinks that that is all you provided. He told me that if I thought you waved some kind of magic wand when you came, I was sadly mistaken. So now I wonder how many in the department there have ever reached this level of understanding. At the time, his telling me this gave me hope because I realized that whatever you did perhaps needed to be done only by a few in the department to have the same kind of positive effect. This conversation occurred before I had my crystallizing experience this spring, so at that point I still wasn't sure how you had done what you had done, but I was sure that you had done something and that I was getting closer to understanding what it was. In order to do what you did, you had to have produced an unusually high number of really good students. So I guess that to reach this level of understanding, one needs to put the student first, but also needs to be caring enough to endure the uncertainty involved in getting to this level.

I understand that by trying out what Charlie Smith did in his calculus classes, I was really just adopting a stance that would allow me to watch students learn, to identify places where they were having trouble, and to try to find ways of helping them over the difficulties. In a way I was adopting a stance that would allow me to improve my teaching. I think that active learning by itself may not provoke substantially more learning by students, but it certainly provides a way for a teacher to improve what he does. And I expect that I will discover other ways of helping students learn in the future. So what Charlie described to me isn't the answer to what mathematicians are doing at Potsdam, but rather it is a method to discover what they are doing. And it certainly provides a framework in which a teacher can find ways to improve his work.

So this is my progress report for the year. This is where I am now. I expect that I will see other things this fall as I watch my students work.